SOLUTION BY A. M. HARDING, University of Arkansas.

$$f'(x) = -e^{-cx}(c + c\cos x + \sin x).$$

$$f''(x) = e^{-cx}(c^2 + c^2\cos x + 2c\sin x - \cos x).$$

Now $f'(x) = 0$ only when $c + c\cos x + \sin x = 0$,
that is, when

$$c(1 + \cos x) + \sin x = 0,$$

or

$$2c\cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} = 0.$$

Hence,

$$\cos\frac{x}{2} = 0, \quad \text{and} \quad x = \pi, 3\pi, 5\pi, \cdots (2n-1)\pi, \cdots$$

or

$$c\cos\frac{x}{2} + \sin\frac{x}{2} = 0, \quad \text{and} \quad x = 2\arctan(-c).$$

When $x = \pi, 3\pi, 5\pi, \cdots, (2n-1)\pi, \cdots, f''(x) = -c(2n-1)\pi$, which is always positive.

Hence, the minimum value of $f(x)$ is obtained by giving x any of these values. When $x = 2\arctan(-c)$, that is, $c\cos x/2 + \sin x/2 = 0$,

$$f''(x) = e^{-cx}[\{c^2(1 + \cos x) + c\sin x\} + c\sin x - \cos x]$$

$$= e^{-cx}\left[2c\cos\frac{x}{2}\left(c\cos\frac{x}{2} + \sin\frac{x}{2}\right) + 2c\sin\frac{x}{2}\cos\frac{x}{2} - \cos x\right]$$

$$= e^{-cx}\left[2c\sin\frac{x}{2}\cos\frac{x}{2} - \cos x\right] = e^{-cx}\left[2\sin\frac{x}{2}\left(-\sin\frac{x}{2}\right) - \cos x\right]$$

$$= -e^{-cx}\left[2\sin^2\frac{x}{2} + \cos x\right] = -e^{-cx}[1 - \cos x + \cos x] = -e^{-cx}.$$

Now $-e^{-cx}$ is negative for the above value of x. Hence the maximum value of $f(x)$ is obtained by giving x the value $2\arctan(-c)$.

Also solved by W. C. EELLS, PAUL CAPRON, H. C. FEEMSTER, G. W. HARTWELL and the PROPOSER.

MECHANICS.

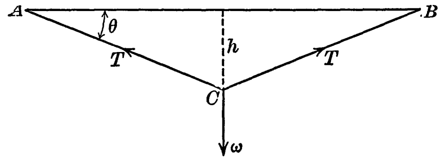
286. Proposed by C. N. SCHMALL, New York City.

A slightly elastic string is just long enough to reach between two hooks on the same horizontal line. A ring of weight w is placed at its middle point. Show that the ring will sink through a distance $h = a\sqrt[3]{3e\omega/2}$, where e is the elasticity of the string and $2a$ the distance between the two hooks.

SOLUTION BY B. F. FINKEL, Drury College.

Since w is at the middle point of the string, the tension T in the two halves of the string is the same when the string is in equilibrium. Let θ be the angle which the string makes with the horizontal line.

Then $AC = a \sec\theta$. Hence the strain in AC is



$$\frac{a \sec\theta - a}{a} = \sec\theta - 1.$$

By Hooke's Law, we have $T/(\sec\theta - 1) = e$. (1)
For equilibrium, we have $2T\sin\theta = w$. (2)
Hence,

$$2e\sin\theta(\sec\theta - 1) = w,$$

or

$$\tan\theta - \sin\theta = \frac{w}{2e}. \tag{3}$$

Since the string is only slightly elastic, θ must be small.

When θ is small we may take $\tan\theta = \theta + (\theta^3/3)$ and $\sin\theta = \theta - (\theta^3/6)$. Hence, substituting these values in (3) and solving for θ, we have $\theta = \sqrt[3]{w/e}$.

But $\tan\theta = h/a$, or $\theta + (\theta^3/3) = h/a$. Substituting for θ and solving for h, we have

$$h = a\left\{\sqrt[3]{\frac{w}{e}} + \frac{w}{3e}\right\}.$$

The proposer's result is incorrect, as the test of physical dimensions shows.

290. Proposed by B. F. FINKEL, Drury College.

A fox, pursued by a hound, is running with uniform velocity over a frail arch in the form of a cycloid: the hound stops at a weak point of the arch, then tumbles through, and reaches the level ground with a velocity equal to that of the fox. Prove that the fox exerted no normal pressure on the arch at the point where the hound fell through. Walton's *Problems in Theoretical Mechanics*, p. 662.

Solution by the Proposer.

Let $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ be the parametric equations of the cycloidal arch and v_0, the speed of the fox. Then at the point where the hound tumbled through, we have the relation $v_0^2 = 2gy_0$, where y_0 is the ordinate of the arch at that point.

Now the centripetal force of the fox is $m(v_0^2/R)$, where m is his mass and R the radius of curvature of the arch. The force exerted by gravity in opposition to this force is $mg\cos\phi$, where ϕ is the direction of the arch at any point.

Hence, the normal pressure on the arch at any point, assuming $m = 1$, is

$$P = \frac{v_0^2}{R} - g\cos\phi.$$

From the equations of the arch, we have

$$\frac{dy}{dx} = \tan\phi = \cot\frac{\theta}{2} = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right).$$